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Physics-Informed Neural Networks for Modelling Problems with Material Interfaces

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Abstract—This paper examines the use of Physics-Informed Neural Networks (PINNs) for solving electromagnetic interface problems, with a particular focus on scenarios involving two distinct materials with differing electromagnetic properties and a shared interface. A representative test case involving concentric disks with differing permeabilities is employed to solve Poisson's equation for the z-component of the magnetic vector potential. The results generated by the PINN are compared to those obtained using the Finite Element Method (FEM) with errors typically below 3.3%. The findings indicate that PINNs are wellsuited for tackling the complexities associated with electromagnetic modeling involving interfaces.

I. INTRODUCTION

In recent years, deep learning and neural networks (NNs) have become dominant methodologies across a wide range of fields. A significant focus of contemporary research is the application of NNs in scientific computing, particularly for solving differential equations that underpin many scientific and engineering challenges [1]. Among these methods, Physics-Informed Neural Networks (PINNs) have gained substantial attention for their effectiveness in this domain [2].

PINNs have shown substantial applicability in addressing electromagnetic problems, particularly in solving Maxwell's equations and other complex phenomena involving electric and magnetic fields. Their flexibility, combined with the ability to incorporate physical laws directly into the training process, makes them highly suitable for tackling the challenges inherent in electromagnetic simulations. In [3], the study investigates the application of PINNs for solving magnetostatic boundary value problems. The findings highlight the critical role of domain similarity in influencing the performance of PINNs, highlighting the need for further research to improve their efficiency in electromagnetic simulations. Another critical aspect of electromagnetic modeling is ensuring continuity at the interface between two different materials. When materials with distinct electromagnetic properties come into contact, accurately predicting field continuity-such as the values of magnetic fields-at these interfaces is essential. Ensuring compatibility at the interface is fundamental to maintaining the physical accuracy of simulations and is crucial for understanding the behavior and performance of electromagnetic devices.

This paper proposes a PINN for solving an electromagnetic problem involving two distinct materials separated by an interface, which is a simple yet representative test case.

II. PROBLEM DESCRIPTION

For our problem we consider the Poisson's equation, as described in equation (1),

$$-\mu^{-1}\Delta a_z(\mathbf{x}) = j_z(\mathbf{x}), \quad \mathbf{x} \in \Omega,$$
(1)
$$a_z(\mathbf{x}) = 0, \quad \mathbf{x} \in \partial\Omega,$$

where a_z represents the z-component of magnetic vector potential, and j_z denotes the excitation current density. Homogeneous Dirichlet boundary conditions are applied at the domain boundary ($\partial\Omega$), where Ω represents the computational domain. The excitation current density (j_z) is $10^6 \frac{A}{m^2}$. The geometry of the problem consists of two concentric disks with an interface between them. The radius of the outer disk is 0.5 m, while the inner disk has a radius of 0.125 m. Each disk is composed of a different material, resulting in distinct permeability μ values for each material. Outer disk has a relative permeability of $\mu_{r_1} = 1$, while the inner disk has a relative permeability of $\mu_{r_2} = 10$. The permeability of free space (μ_0) is taken as $4\pi \times 10^{-7} \frac{\text{H}}{\text{m}}$.

III. PROBLEM IMPLEMENTATION

Each disk is assigned a separate PINN for training. Each PINN incorporates several loss terms, which are outlined as follows:

$$L_{Total}^{Out} = \lambda_1 L_{Physics}^{Out} + \lambda_2 L_{Boundary}^{Out} + \lambda_3 L_{Interface}^{Out}$$
(2)

$$L_{Total}^{In} = \lambda_4 L_{Physics}^{In} + \lambda_5 L_{Interface}^{In}$$
(3)

For each disk, a physics loss function is defined based on Poisson's equation. The outer disk includes an additional boundary condition loss term. Both disks share an interface loss term, which couples the two NNs together. The loss terms in each region are associated with weight factors, denoted as λ , which are multiplied by their respective loss terms and are predetermined prior to training. The transmission conditions at the interface (Γ) are described below.

$$n \times (h_2 - h_1) \bigg|_{\Gamma} = 0 \tag{4}$$

$$n \cdot (b_2 - b_1) \bigg|_{\Gamma} = 0 \tag{5}$$

Where n represents the normal component, and h and b denote the magnetic field intensity and magnetic flux density, respectively. The conditions integrated into the interface loss terms are formulated based on the primary conditions described in equations (4) and (5), and are presented as follows:

$$\mu_2^{-1}\left(\frac{\partial a_{z_2}}{\partial n}\right)\Big|_{\Gamma} = \mu_1^{-1}\left(\frac{\partial a_{z_1}}{\partial n}\right)\Big|_{\Gamma} \tag{6}$$

$$a_{z_2} \Big|_{\Gamma} = a_{z_1} \Big|_{\Gamma} \tag{7}$$

The neural network architectures employed are fully connected feedforward networks. The input layer is designed to accept two features—coordinates x and y. In the final stage, which is the testing stage of the PINN, the predictions are compared with the results obtained from the Finite Element Method (FEM) analysis implemented in the ONELAB software [4].

IV. RESULTS

Figure 1 compares the z-component of the magnetic vector potential (a_z) obtained using the FEM and PINN. Fig. 1-(a) shows the FEM results, which serve as the reference, with a_z values ranging from approximately 0 to 0.12 $\frac{Wb}{m}$. Fig. 1-(b) presents the PINN predictions, which closely match the FEM solution, indicating the PINN's capability to learn the underlying physics governing a_z within the domain. Fig. 1-(c) depicts the error distribution between the FEM and PINN results, with absolute values ranging from about 0.0028 to 0.0040 $\frac{Wb}{m}$. Notably, slightly higher discrepancies are observed near the central region and along the boundaries, with the maximum error amounting to about 3.33%.

Figure 2 presents PINN model predictions for the interface conditions against the reference solution. The inner disk interface and outer disk interface predictions align closely with the reference curve, demonstrating the PINN's accuracy in capturing the magnetic vector potential across interfaces.

V. CONCLUSION

This study has demonstrated the potential of PINNs to model electromagnetic interface problems with reasonable accuracy compared to traditional FEM. By incorporating physical laws directly into the network's training process, the PINN was able to capture the behavior of magnetic vector potential across different materials, showing errors generally below 3.3% compared to FEM results. Further validation and extension of the method to more complex geometries and materials are necessary for broader practical adoption.

REFERENCES

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Fig. 1. Comparison of the z-component of the magnetic vector potential results: (a) FEM results, (b) PINN predictions, and (c) absolute error between FEM and PINN solutions.



Fig. 2. Comparison of interface predictions with reference values for the inner and outer disks.